<u>“Struck by Lightning” Supplementary Materials</u>

<u>Reminder About Solution to Gambler’s Ruin Problem.</u>

<u>Recall</u> that we were considering the following game (to be referred to as the <u>original game</u>). **A** starts with a pennies, and **B** starts with $8-a$ pennies. A fair 6-sided die is repeatedly rolled. If it comes up 1 or 2, then **B** gives one penny to **A**. If it comes up 3, 4, 5, or 6, then **A** gives one penny to **B**. This is repeated until either **A** or **B** wins all the pennies. That person is the “winner”. Recall that we wrote $s(a)$ for the chance that **A** wins this game, starting with a pennies.

What follows is a very brief “reminder” of how we derived a formula for $s(a)$. Learn it well; we will come back to these issues soon!

Step #1. Obviously $s(0)=0$ and $s(8)=1$.

Step #2. By considering what happens on the *first* bet, we see that

$$s(a)=\frac{1}{3}s(a+1)+\frac{2}{3}s(a-1)\,,$$

for $a=1,2,3,4,5,6,7$.

Step #3. Since $s(a)=\frac{1}{3}s(a)+\frac{2}{3}s(a)$, this last formula can be re-written as

$$s(a+1)-s(a)=2\left[s(a)-s(a-1)\right]\,.$$

Step #4. Hence, setting $x=s(1)$, we see that

$$s(1)-s(0)=x,\quad s(2)-s(1)=2x,\quad s(3)-s(2)=4x,\quad \text{etc.}$$

and in general that

$$s(a+1)-s(a)=2^a x$$

for $a=0,1,2,3,4,5,6,7$.

Step #5. It follows that, for $a=0,1,2,3,4,5,6,7,8$,

$$\begin{aligned} s(a)=s(a)-s(0) &= (s(a)-s(a-1))+(s(a-1)-s(a-2))+\ldots+(s(1)-s(0)) \\ &= \left(2^{a-1}+2^{a-2}+\ldots+2+1\right)x=(2^a-1)\,x\,. \end{aligned}$$

Step #6. Since $s(8)=1$, it follows that $x=\frac{1}{2^8-1}$, whence

$$s(a)=\frac{2^a-1}{2^8-1}=\frac{2^a-1}{255}\,.$$

So, for example, $s(6)=(2^6-1)/255=63/255\doteq 0.2470588\doteq 24.7\%$.