Probability and Justice

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(National Judicial Institute meeting, Vancouver, Feb 21, 2024)

About Me . . .

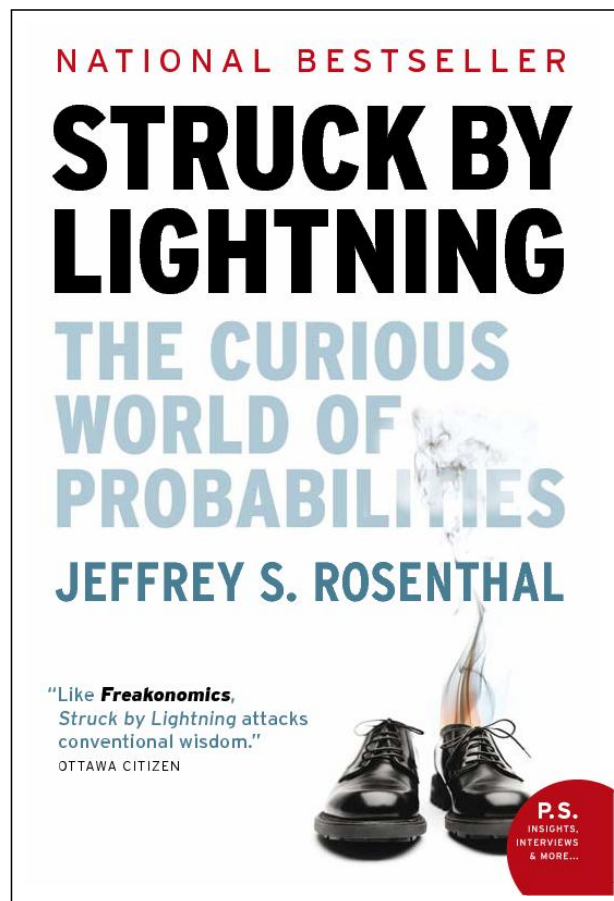I'm a Professor of Statistics. A typical day's work:

Therefore
$$\alpha^-(\beta_i^{(d)}) = \alpha^+(\beta_i^{(d)} - \ell/\sqrt{d})$$

$$\stackrel{d^{1/2}}{\approx} \alpha^+(\beta_i^{(d)}) - \frac{(\ell(\beta_i^{(d)})I^{1/2}(\beta_i^{(d)}))'}{2}\left(\frac{-\ell}{\sqrt{d}}\right)\phi\left(-\frac{I^{1/2}(\beta_i^{(d)})\ell}{2} - \frac{\epsilon\ell K'''(\beta_i^{(d)})}{6I^{1/2}(\beta_i^{(d)})}\right)$$

$$- \exp(-\epsilon\ell^2(\beta_i^{(d)})K'''(\beta_i)/6)\frac{(\ell(\beta_i^{(d)})I^{1/2}(\beta_i^{(d)}))'}{2}\left(\frac{-\ell}{\sqrt{d}}\right) \times$$

$$\times\phi\left(-\frac{I^{1/2}(\beta_i^{(d)})\ell}{2} + \frac{\epsilon\ell K'''(\beta_i^{(d)})}{6I^{1/2}(\beta_i^{(d)})}\right)$$

Then, since $\underline{\ell} \stackrel{d^{1/2}}{\approx} \ell + \underline{\epsilon}\ell' \stackrel{d^{1/2}}{\approx} \ell + \epsilon\ell' = \ell + \frac{\ell\ell'}{d^{1/2}}$, we have that

$$\mu(\beta_i^{(d)}) \stackrel{d^{1/2}}{\approx} \frac{1}{2d^{1/2}}\left[-\alpha^+\ell + \left(\ell + \frac{\ell\ell'}{d^{1/2}}\right) \times\right.$$

$$\left(\alpha^+(\beta_i^{(d)}) - \frac{(\ell(\beta_i^{(d)})I^{1/2}(\beta_i^{(d)}))'}{2}\left(\frac{-\ell}{\sqrt{d}}\right)\phi\left(-\frac{I^{1/2}(\beta_i^{(d)})\ell}{2} - \frac{\epsilon\ell K'''(\beta_i^{(d)})}{6I^{1/2}(\beta_i^{(d)})}\right)-\right.$$

And then one day I wrote a successful book . . .

NATIONAL BESTSELLER

STRUCK BY LIGHTNING

THE CURIOUS WORLD OF PROBABILITIES

JEFFREY S. ROSENTHAL

"Like **Freakonomics**, *Struck by Lightning* attacks conventional wisdom."
OTTAWA CITIZEN

P.S.
INSIGHTS,
INTERVIEWS
& MORE...

Then I was interviewed by the media about: Opinion Polls . . .

TORONTO STAR
www.thestar.com

Use caution when approaching polls

Party preference numbers have varied

But differences deceiving, experts say

Jan. 21, 2006. 01:00 AM

ROBERT BENZIE

QUEEN'S PARK BUREAU CHIEF

Not a vote has been counted. The party leaders are still criss-crossing the country in a frantic final bid for support. Campaign volunteers are working the phones, replacing lawn signs and arranging election day transportation for voters who require it.

And yet we already know — or at least we think we know — that on Monday Stephen Harper's Conservatives will defeat Paul Martin's Liberals, and the NDP's Jack Layton will be the federalist leader holding the balance of power in a minority Parliament.

How have we concluded this? Well, the polls tell us so.

But don't different major polls all seem to have differing results?

Yes and no, says Jeffrey Rosenthal, professor of probability theory at the University of Toronto and a leading expert on polling.

Rosenthal, author of the recent bestseller *Struck by Lightning: The Curious World of Probabilities*, says "any one poll should always be taken with a grain of salt.

Crime statistics . . .

You're safer than you think: Statistics expert

Article   Photos (4)

Published On Sat Jan 19 2008

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Paola Loriggio
Staff Reporter

As long as there have been cities, there has been fear. Fear of violence, fear of death, fear of anonymous, big city crime.

High-profile cases of random crime — like the recent shootings of John O'Keefe and Hou Chang Mao, both innocent bystanders killed within a week — feed the public's anxieties.

Police cordon Gerrard St. E. near Broadview Ave. as part of the investigation into a fatal shooting Thursday. A statistics expert says the odds are better of dying in a car accident, than being killed by random crime.

HENRY STANCU/TORONTO STAR FILE PHOTO

But is that fear justified? Random crime isn't going away, but neither is it increasing. Does a spate of random killings put us in greater danger than before? The *Star* asked an expert statistician to assess the risk.

University of Toronto professor Jeffrey S. Rosenthal is the author of *Struck*

Pedestrian death counts . . .

Not so rare for rarities to occur in waves: Professor

TORSTAR NEWS SERVICE
Published: January 29, 2010 5:23 a.m.
Last modified: January 29, 2010 12:40 a.m.

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Scary numbers have dominated Toronto headlines this month: Seven dead in seven days.

Fourteen pedestrians killed across the GTA. The deadliest January for city pedestrians in a decade.

No wonder people are walking scared.

31.9

► Seven isn't that big a number when looked at through a statistician's lens. Jeffrey Rosenthal calculates that between 2000 and 2009, Toronto witnessed an average of 31.9 pedestrian deaths per year and 2.7 deaths per month. Using Poisson distribution, this means there is about a 1.9 per cent chance of there being seven or more pedestrian deaths in a single month.

Sports . . .

Including Very High-Profile Sports Moments . . .

And lottery jackpots . . .

TORONTO STAR
www.thestar.com

Advice for lottery winners: Chill out

Apr. 30, 2006. 10:14 AM

ELLEN ROSEMAN

What are your odds of winning the lottery?

Unimaginably small, says University of Toronto statistics professor Jeffrey Rosenthal.

You have one chance in 14 million to score big playing Lotto 6/49. (That's where you have to match all six numbers chosen from 1 to 49.)

"To put it in context, you are over 1,000 times more likely to die in a car crash within the year," Rosenthal says in his book, *Struck by Lightning: The Curious World of Probabilities* (HarperCollins, $34.95).

"In fact, you are more likely to die in a car crash on your way to the store to buy your lottery ticket than you are to win the lottery jackpot.

Including the Lottery Retailer Scandal
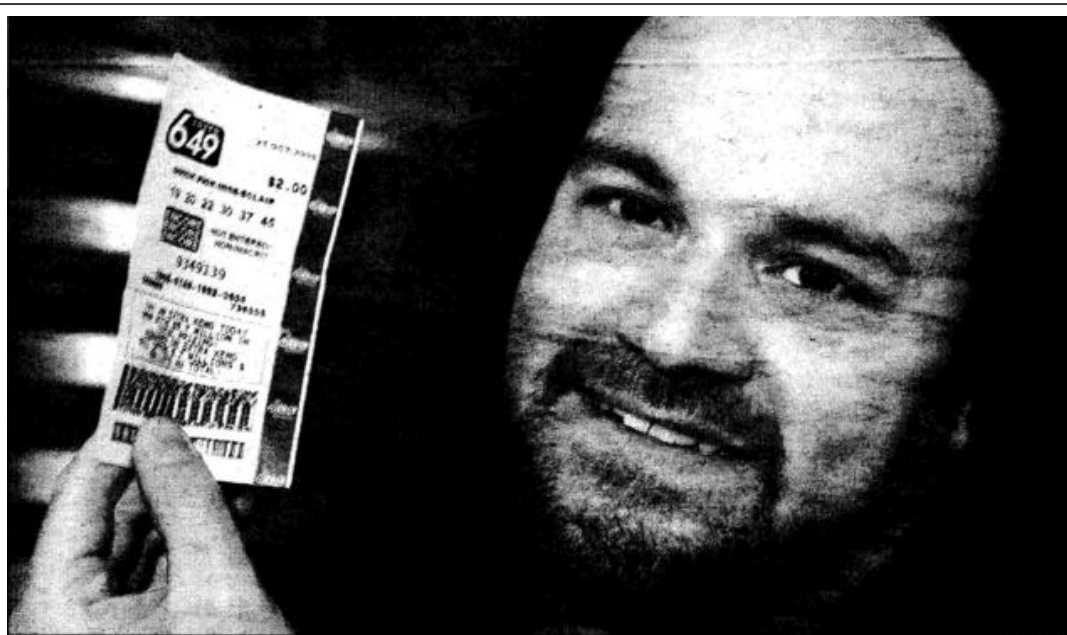
University of Toronto statistician Prof. Jeffrey Rosenthal holds up a 6/49 ticket. He says so many retailers have won prizes it raises suspicions.

Storekeepers lucky?

Lottery officials dismiss statistician's claim that retailers' wins defy odds

Which Had Serious Consequences

Ontario Lottery chief fired

Mar. 24, 2007, 6:12 a.m.   5 comments

LOTTERY POST

O L G

Ontario Lottery and Gaming Corporation: Ontario Lottery chief fired

E-mail to a friend

Printer-friendly

Link to this story

Rating: ★★★★☆

anonymity.

The head of the Ontario Lottery and Gaming Corporation was dismissed from the scandal-plagued organization on Friday, according to CBC News.

CEO Duncan Brown was escorted out of the lottery corporation's offices in Toronto, two sources told the CBC, speaking on condition of

. . . Which Spread to B.C., Too

B.C. lottery boss terminated

Last Updated: Friday, June 1, 2007 | 6:43 PM ET
CBC News

The president and CEO of the B.C. Lottery Corporation has been fired following a scathing report by the province's ombudsman that found ticket retailers were winning too often.

In a written statement, the lottery corporation's board said it was terminating Vic Poleschuk effective immediately.

And Led to Criminal Charges

$12.5M lottery prize theft leads to 3 arrests

Last Updated: Wednesday, September 29, 2010 | 10:25 PM ET

Comments 462   Recommend 322

CBC News

The case of Kathleen Chung, who allegedly cashed the winning ticket at her brother's convenience store in Burlington in early 2004, was profiled by the CBC's Fifth Estate, triggering a report by the Ontario ombudsman. (CBC)

Three family members in the Toronto area have been charged in the theft of a $12.5-million lottery prize, while police seek the rightful owner of the Lotto Super 7 ticket bought in 2003.

Two of the accused are a father and son who worked at a Burlington lottery outlet and were actively stealing tickets from customers, Ontario Provincial Police Commissioner Chris Lewis

. . . And Millions of Dollars Repaid!

Lottery fraud victims claim $12.5M prize – plus interest

Seven men with wraparound smiles claimed their seven-year-old $12.5 million lottery win Thursday, which stands now at $14.85 million.

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TANNIS TOOHEY / TORONTO STAR

http://probability.ca/lotteryscandal/

So what is the connection to JUSTICE?

Statistics and Justice both involve evaluating evidence.

Justice:

"beyond a reasonable doubt"

"balance of probabilities"

"preponderance of the evidence"

Statistics:

"statistically significant"

"the probability is more than X"

"the p-value is less than Y"

"19 times out of 20" / "95% certain" / "99.9% certain"

Are they related?   (Oct 2013 CIAJ talk – thanks Paul Schabas!)

Does it Matter?

The New York Times   April 20, 2019

A Leading Cause for Wrongful Convictions: Experts Overstating Forensic Results

These three men spent decades in prison as a result of statistical exaggerations. They were among 150 men and women released from prison after their wrongful convictions were overturned in 2018.

"An expert can say whatever they want," said Simon Cole, the director of the registry and a professor of Criminology, Law and Society at UC -Irvine.

That includes offering up invented odds like "one in a million" or "1 in 129,600," the registry says.

How Statisticians Weigh Evidence

<u>Example:</u> Can your friend distinguish Coke from Pepsi? Do a test!
Guesses right the first time. Proof of ability? No, could be luck!
What about twice in a row? three times? ten times?

The <u>p-value</u> is the probability of such a result <u>if</u> it's just random.

- Guess right <u>once</u>: p-value $= 1/2 = 50\%$.
- Guess right <u>twice</u> in a row: p-value $= (1/2) \times (1/2) = 25\%$.
- Guess right <u>five times</u> in a row: multiply ("independent"):

p-value $= (1/2) \times (1/2) \times (1/2) \times (1/2) \times (1/2) \doteq 3.1\%$.

The smaller the p-value, the more it seems to "prove" something.
Usual standard: "significant" if p-value less than 5% (i.e., 1 in 20).

For Coke versus Pepsi: two in a row <u>not</u> significant, five in a row <u>is</u>.

<u>Similarly:</u> Disease with 50% fatality rate. New drug: does it work?
If it saves 5 patients in a row, then yes it's "significant".

Important! Useful! Widely used! But potentially problematic . . .

To multiply or not to multiply?

Guess correctly by chance, <u>once</u>: Prob $= 1/2$.

Guess correctly by chance, <u>twice:</u> Prob $= (1/2) \times (1/2) = 1/4$.

Correct? Yes, since two guesses by chance are <u>independent</u>.

(Related to that joke about a bomb on an airplane . . .)

But multiplying might not <u>always</u> be valid.

Example: The United States and NFL (American) football:
(1) 50.8% of Americans are female.
(2) 64% of Americans watch NFL football (survey).

So, what percentage are females who watch NFL football?

Is it $(50.8\%) \times (64\%) = 32.5\%$?

No, it's actually just 27.9%. Why? Because "only" 55% of U.S.
women (and 73% of men) watch NFL football.

Not independent! Can't multiply in this case! Be careful!

"Out of How Many?"

True story: Ran into my father's cousin at Disney World!

Surprise! One chance in 230,000,000? Deep significance?

But wait. We saw several thousand people there.

And, we would have been surprised by hundreds of people.

It follows that some such meeting had about one chance in 200.

Might well happen over a lifetime. (Has it to you?)

TV Interview: Reunited Half-Brothers

On a beach, brotherhood
Chance meeting in Hawaii brings half siblings together

Brothers Rick Hill and Joe Parker met by chance on a beach in Hawaii.

By Brian R. Ballou
Globe Staff / April 28, 2011

E-mail | Print | Reprints | Text size − +

Waikiki Beach wasn't part of Rick Hill's vacation plans last Monday, but the
Lunenburg resident and his family decided to make a quick stop.

Striking? Yes. Deeper "meaning"? Or just chance?

Out of how many other estranged Americans? (28%?)

TV: One success out of so many chances is luck, not "meaning".

An Old Legal Case: Malcolm Collins (California)

- On June 18, 1964, in Los Angeles, an elderly lady was pushed down in an alley, and her purse was stolen.

- Witnesses said: a young Caucasian woman, with a dark blond ponytail, ran away with the purse, into a yellow car, which was driven by a Black man, who had a beard and moustache.

- Four days later, Malcolm and Janet Collins were arrested, because they fit these characteristics (mostly).

- At trial, the prosecutor called "a mathematics instructor at a nearby state college" (Daniel Martinez). The prosecutor told the mathematician to assume certain ("conservative") probabilities:
  - Black man with a beard: 1 out of 10
  - Man with moustache: 1 out of 4
  - White woman with blonde hair: 1 out of 3
  - Woman with a ponytail: 1 out of 10
  - Interracial couple in car: 1 out of 1,000
  - Yellow car : 1 out of 10

The mathematician then computed the probability that a random couple would satisfy all of these criteria, by multiplying:

$(1/10) \times (1/4) \times (1/3) \times (1/10) \times (1/1000) \times (1/10) = 1/12,000,000$

Was this reasoning valid?

- The facts? No, these individual probabilities were just <u>assumed</u>.

- Multiplying? No! If have a beard, then moustache more likely!

Similarly, if have Black man and White woman, then <u>of course</u> have an Interracial couple!

- Correctly interpreted probability? No! Remember "out of how many"! Los Angeles County 1964 "suspect population": 6,537,000. So, the probability of two such couples is quite large.

- Collins was convicted at trial, based on this 1/12M probability.

- Acquitted on appeal, Supreme Court of California, 1968:

". . . the testimony as to mathematical probability infected the case with fatal error and distorted the jury's traditional role of determining guilt or innocence".

A More Serious Legal Case: Sally Clark (England)

- Solicitor in Cheshire, England.

- Had two sons; each died in infancy.

- "cot death" (SIDS)? Or murder!?!

- 1999 testimony by paediatrician Sir Roy Meadow: "the odds against two cot deaths in the same family are 73 million to one".

- Convicted! Jailed! Vilified! Third son temporarily taken away!

Was "73 million to one" computed correctly? And, was it the right thing to compute? No!

How did Meadow compute that "73 million to one"?

He said the probability of <u>one</u> child dying of SIDS was one in 8,543, so for <u>two</u> children dying, we <u>multiply</u>: $(1/8,543) \times (1/8,543) = 1/72,982,849 \approx 1/73,000,000$.

Clark Case: Valid Probability Calculation?

Was the multiplication valid? No! SIDS tends to run in families, so once a family has had one SIDS case, the second one is <u>more likely</u>.

Were the probabilities accurate? No! He neglected factors which <u>increase</u> the probability, e.g. twice as likely for boys. (1/170,000?)

Was the interpretation valid? No! What about "out of how many"? (Millions of families in the U.K. / World!)

"Prosecutor's Fallacy": conflating two <u>different</u> probabilities.

Royal Statistical Society: "approach is . . . statistically invalid"

- Clark was eventually acquitted, on second appeal. (then died)

- The U.K. General Medical Council ruled that Meadow's evidence was "misleading and incorrect", constituting "serious professional misconduct". He was barred from future court work.

- Several other people's convictions were overturned on appeal.

- Prosecutors/judges everywhere learned a valuable lesson. (?)

A Related Case: Waneta & Tim Hoyt (New York)

Had <u>five</u> babies in 1965 – 1971. <u>All</u> died.
Ages (months): 3, 28, 1.5, 2.5, 2.5.

Pediatrician Alfred Steinschneider investigated,
wrote 1972 article for journal *Pediatrics*.
Conclusion: "genetically-linked SIDS".

In 1977, they were allowed to adopt a son,
who survived to adulthood.

In 1985, some prosecutors and pathologists got suspicious, and
investigated. Eventually, Waneta Hoyt confessed to suffocating all
five children, to stop them from crying.

She later "recanted" her confession, but was still convicted in 1985
of five murders. She died in prison in 1998 (age 52).

So, sometimes statistical evidence <u>is</u> indicative, even when you
can't just multiply. It shouldn't necessarily be ignored.

A Nursing Case: Lucia de Berk (Netherlands)

- Hospital nurse in The Hague, Netherlands.

- Arrested for several murders and attempted
murders, after discovery that she was on duty
for 14 of 27 (51.9%) "incidents" (i.e. deaths,
or near-deaths), despite working just 203 out
of 2,694 (7.5%) shifts in her three wards.

- Prosecution (2003): one chance in 342
million of this occurring by chance alone!

- Accurate facts? Some controversy whether all these incidents
had actually taken place <u>during</u> de Berk's shifts (versus just before
or after), and whether definition of "incident" was adjusted *post
hoc*. Also, she was assigned to many elderly/terminal patients.

- Valid calculation? Many statisticians thought no.

- Accurate interpretation? No! What about "Out of how many"?
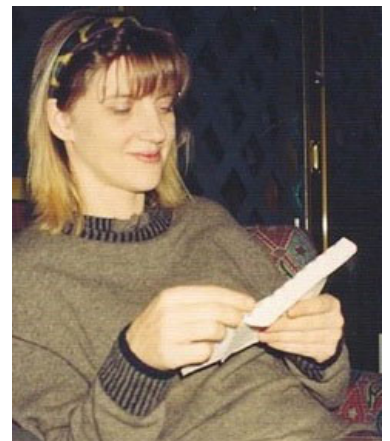
- The prosecution statistician, Henk Elffers, <u>had</u> tried to account for "out of how many", by multiplying by 27 (the number of nurses in one of the hospitals).

- Is that sufficient? Surely not! Many more nurses <u>somewhere</u> in the Netherlands / World. Multiply by <u>all</u> of them?

- Statistician: "the data are used twice: first to identify the suspect, and again in the computation of Elffers' probabilities".

- de Berk was convicted of multiple murders and attempted murders in March 2003, primarily on the basis of "1 in 342 million".

- The convictions were upheld on appeal, June 2004, mostly on <u>other</u> grounds: elevated digoxin levels in some of the corpses.

- October 2007 Dutch "Posthumus II Commission" report: "the hypothesis of digoxin poisoning was disproven [through new testing; similar to Susan Nelles case], the statistical data were biased and the analysis incorrect, and the conclusions drawn from it invalid."

- Case reopened June 2008. Not guilty verdict, April 2010.

The Cases Keep Coming: Kathleen Folbigg (Australia)

The Washington Post | Published June 4, 2023

Mother who served 20 years in deaths of 4 children freed after new evidence

Her trial in 2003 focused on her diary entries, in which Folbigg — now in her 50s — wrote she had "failed as a mother, a woman." Prosecutors argued that the deaths of four young children in a row could not be a tragic coincidence, and she was excoriated in the media. A jury convicted her of

In 2021, dozens of scientists — including two Nobel laureates — signed a petition urging the governor of New South Wales to pardon Folbigg, arguing that she was "wrongfully incarcerated" and that genetics may have caused the deaths. Geneticists have found rare mutations in the DNA of Folbigg and her daughters that can cause sudden death in infancy and childhood, and other variants found in her sons' DNA have also been connected to deaths in young children.

British nurse Lucy Letby imprisoned for life for murders of seven babies, attempted murders of six

A former neonatal nurse convicted of murdering seven babies in her care and trying to kill six others at a hospital in northern England was sentenced Monday to life in prison with no chance of release by a judge who said she was cruel, cunning and callous, and acted with "malevolence bordering sadism."

Ms. Letby sickened babies by injecting intravenous lines with air, poisoning some with insulin and force-feeding others milk. After killing them, she sometimes sobbed in grief, made keepsakes for parents and bathed the little bodies and dressed them for burial.

Also planned is an independent inquiry into what happened at the Countess of Chester Hospital and how staff and management responded to the spike in neonatal unit deaths.

RESPONSE AT: rexvlucyletby2023.com

The case against Lucy Letby lacked scientific evidence and is based on unverified hypotheses

The importance of the underlying physiology of premature neonates was not properly highlighted by experts

A Case I was Involved With: Leighton Hay

Accused of being 2002 murder accomplice.

Witnesses: Hair was "two inch picky dreads".

But Hay was shaved nearly bald when arrested.

Crown: He <u>shaved his head</u> after the crime!

Evidence: Tiny hair clippings in a garbage bin and on an electric shaver in his home.

Convicted in 2004. Fresh appeal in 2011.

Question: Were those clippings from a <u>scalp</u>?

Statistical data: <u>Thickness</u> of the clippings.

Fact: Scalp hairs are usually ≤ 125 microns thick, but beard hairs are often thicker. So what did that tell us?

My expert report: Of the 368 clippings collected, the number from a scalp was between 0 and 106 (29%), with the rest from a beard.

2013 SCC 61 judgment: New trial granted. Hay released from jail.
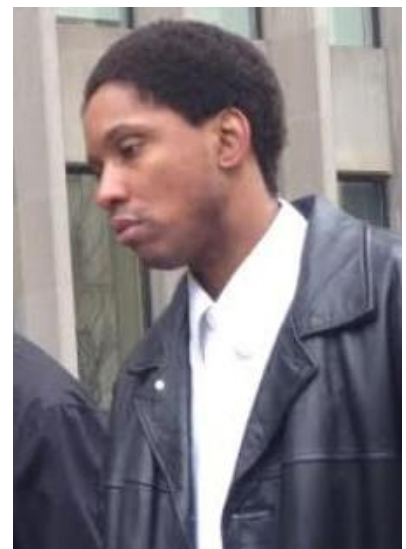
Another Case I was Involved With: Yuk Yuen Lee

Accused in 2013 of running a marijuana grow-up in Toronto.

Police seized $1378 + 2240$ plants, all claimed to be marijuana.

However, they only actually <u>tested</u> $2 + 1$ of them (!).

Convicted at trial, but what about the <u>sentence</u>?

If more than 500 plants, then mandatory three-year jail term.

My expert report: The testing was only sufficient to statistically conclude that at least $138 + 16$ of the plants were marijuana.

2017 ONSC 2403 judgment: "Crown counsel took issue with respect to Professor Rosenthal's credibility. . . . I did not find Professor Rosenthal lacking in credibility. His evidence was offered in the manner that one expects of an expert. . . . I do not accept that the Crown has established the number of marijuana plants, thereby allowing the Crown to rely on the mandatory minimum."

Final sentence: Just the time already served in jail.

Statistics and Justice: Reflections

- Statistics and probability have many important applications.
- Including to issues of law and justice.
- They are sometimes misused, to wrongly imply guilt.
- But they can also correctly clarify or refute evidence.
- Probabilities should not be rejected out of hand.
- Rather, the ways they were computed should be carefully scrutinised, with miscalculations corrected, and flaws exposed.
- Including asking such questions as "Accurate Facts?", "Out of How Many?", "To Multiply or Not To Multiply?", etc.
- Hopefully by <u>unbiased</u> statistical experts. (Adversarial system?)

Statistical analysis <u>can</u> sometimes help achieve justice.

But it must be used with caution!   Web: www.probability.ca

Article www.probability.ca/justice ; book <u>Knock On Wood</u> (ch. 19).

Email: jeff@math.toronto.edu   Twitter: @ProbabilityProf