Probability questions for high school students

Let k be a positive integer, and let p be a real number strictly between 0 and 1.

Suppose we start with $\$k$, and repeatedly make independent \$1 bets, where for each bet we have probability p of winning \$1 and probability $1 - p$ of losing \$1.

Say that we "go broke" on a certain bet if we lose all of our money on that bet. (If we do go broke, then we immediately stop betting.)

1. (easy) If $k = 3$, then what is the probability we go broke on exactly the third bet? (Your answer should depend on p.)

2. (medium) If $k = 3$, then what is the probability we go broke on exactly the fifth bet? (Your answer should depend on p.)

3. (hard) Let $r(k, p)$ stand for the probability that we *eventually* go broke, if we start with $\$k$ and have probability p of winning each bet.

(a) Show that $r(k, p) = (f(p))^k$, for some function $f(p)$. [Hint: Think about what $r(k, p)$ really means.]

(b) Find a formula for $f(p)$. [Hint: Relate $r(k, p)$ to $r(k + 1, p)$ and $r(k - 1, p)$.]

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