STA 198F, Fall 2020: Probabilities Everywhere

Class activities for Week 6

Class Poll discussion: Tabulate all of the results of our poll. Based on that, what do you think is the true fraction of all first-year U of T students who would answer "Yes" to the question? Why? What range are you "confident" contains the true fraction?

Whole-class homework discussion: As a group, we will discuss last week's homework readings and questions. Be sure to participate actively, and raise your hand often!

Group "Craps" Exercise:

The game "craps" is played as follows. A player rolls two fair six-sided dice. If their sum is 7 or 11, the player wins right away. If their sum is 2, 3, or 12, the player loses right away. If their sum is any other number, this number becomes the "point". The player then repeatedly rolls two dice until their sum equals the point (in which case the player wins) or equals 7 (in which case the player loses).

1. Make sure everyone in your group understands the rules of craps.

2. As a group, make your best guess of the probability of winning at craps. (You will later tell the instructor.)

Next, compute the probability of winning at craps, as follows:

3. Compute the probability p_i of rolling the sum i on the *first* roll of the two dice, for $i = 2, 3, 4, \ldots, 12$. (Hint: you might want to first make a 6×6 chart of all possible pairs of die values.)

4. Compute the probability q_i that you win at craps *if* we get the result i on the first roll of the two dice, for $i = 2, 3, 4, \ldots, 12$. [Hint: For example, $q_2 = 0$ because if you roll a 2 the first time then you lose. Similarly $q_7 = 1$. Thus, q_2, q_3, q_7, q_{11}, and q_{12} are all easy. The other values of q_i are harder. For example, q_4 is the probability that, if you repeatedly roll two dice, you will get 4 before you get 7, ignoring any rolls that are neither 4 nor 7. How can you compute this probability?]

5. Now that you know all the values of p_i and q_i, compute the overall probability, p, of winning at craps. [Hint: This is just a question of combining all the p_i and q_i together in the right way. Remember that if you do win at craps, you had to get *some* result on the first roll, and then win after that. So, what is the final formula?]

6. How does this probability p (of winning at craps) compare to 50%? Why is this an important question? [Hint: pretend you're at a casino, and play craps repeatedly.]

7. Suppose that in one out of every n games, you could *cheat* at craps so that you will win for sure (instead of just having a certain probability of winning). For what values of n would this allow you to make a long-run profit by repeatedly playing craps? Explain.

Homework assignment:

1. Find at least two public opinion polls about the upcoming (Nov. 3) U.S. presidential election. For each poll, note (if available): (a) the company that conducted the poll; (b) the date(s) on which the poll was taken; (c) the number of people surveyed; (d) the claimed percentage of support of each candidate; and (e) the claimed margin of error. Does the claimed margin of error agree with the formula from our readings?

Then, read the book from the top of page 87 to the end of page 95. While you read, consider (and make brief notes about) the following questions:

2. Summarise the two decisions made in the story "Walk or Ride?" Do you agree with the decisions? Why or why not?
3. What are utility functions, and how can they be useful?
4. How were utility functions used in the book to decide about the wedding, and about phoning "Juan"? (Explain the calculations used.)
5. Summarise the calculations in the book about the insurance policy. Why does the book say that insurance can sometimes be a win-win situation? Do you personally believe in purchasing insurance? Why or why not?
6. Summarise at least two of the various examples of differing utility functions in the section "Your Utility or Mine?".
7. Give an example from your own life where two people had significantly different utility functions, and a conflict that arose as a result.
8. On a scale where seeing a pretty good movie equals +10, decide on the value of your own personal utility function for each of the following events: (a) getting caught in the rain without a jacket; (b) finding a $20 bill; (c) losing a $20 bill; (d) winning one million dollars; (e) winning ten million dollars.
9. Based solely on your utility function values, would you be willing to run through the rain without a jacket (a) in order to pick up a $20 bill you found? (b) in order to pick up a lottery ticket that had one chance in a million of being worth ten million dollars? (Explain your calculations.)
10. Do you personally believe that expected values of utility functions are a good way to make decisions? Do you think that most people make decisions this way (whether they realise it or not)? Why or why not?