Name: ______________________________ Student Number: ________________

STA 447 / 2006 (circle one), Winter 2012, Mid-Term Test

(February 16, 2012. Duration: 60 minutes. Questions: 2. Pages: 3. Total points: 36.)

Notes: You should fully explain your answers! Continue on the back if necessary. NO AIDS ALLOWED. You may use theorems from class. Point values in [square-brackets].

1. Consider a Markov chain with state space $S = \{1, 2\}$, and transition probabilities $p_{11} = 2/3$, $p_{12} = 1/3$, $p_{21} = 1/4$, and $p_{22} = 3/4$.

(a) [3] Compute $p_{12}^{(2)}$.

(b) [2] Determine whether or not this chain is irreducible.

(c) [4] Let $\pi_1 = 3/7$ and $\pi_2 = 4/7$. Prove that $\{\pi_i\}$ is a stationary probability distribution for this chain.

1. (cont'd) Recall that $S = \{1, 2\}$, $p_{11} = 2/3$, $p_{12} = 1/3$, $p_{21} = 1/4$, and $p_{22} = 3/4$.

(d) [3] Determine whether or not $f_{11} = 1$.

(e) [3] Determine whether or not $f_{21} = 1$.

(f) [3] Determine whether or not $\sum_{n=1}^{\infty} p_{21}^{(n)} = \infty$.

(g) [3] Determine whether or not this chain is aperiodic.

(h) [2] Determine whether or not $\lim_{n\to\infty} p_{12}^{(n)} = \pi_2$.

2. Consider a Markov chain with state space $S = \{1, 2, 3\}$, and transition probabilities $p_{11} = p_{12} = p_{22} = p_{23} = p_{32} = p_{33} = 1/2$, with $p_{ij} = 0$ otherwise.

(a) [3] Determine whether or not this chain is irreducible.

(b) [4] Compute f_{ii} for each $i \in S$.

(c) [3] Specify which states are recurrent, and which states are transient.

(d) [3] Compute the value of f_{13}.

[END]