STA261 EXTRA QUESTIONS, SPRING 2004

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Note: What follows are a few extra homework questions, about topics for which the textbook doesn't have enough appropriate questions.

1. Consider the statistical model where $S = [0,1]$, $\Omega = \{1,2\}$, P_1 has density $f_1(s) = 2s$, and P_2 has density $f_2(s) = 3s^2$. Suppose we observe a single observation $s \in S$.

(a) Compute the likelihood function for this model.

(b) Compute the MLE, $\hat{\theta}$, for θ.

(c) Compute $\text{Bias}_\theta(\hat{\theta})$, the bias of $\hat{\theta}$.

2. Consider the statistical model where $S = \{0, 1, 2, \ldots\}$, $\Omega = (0, \infty)$, and for $\theta \in \Omega$, P_θ is the Poisson(θ) distribution, so $P_\theta(s) = e^{-\theta}\theta^s \,/\, s!$. Suppose we observe a single observation $s \in S$. (You may assume that $s > 0$ for simplicity.) Repeat parts (a) to (c) of the previous question. [Hint: For (b), use the Score Equation.]

3. Compute $MSE_\theta(\hat{\theta})$, the mean squared error of $\hat{\theta}$, for each of the two models in the two previous questions.

4. Suppose P_θ has mean θ and variance 1, and we observe $x_1, \ldots, x_n$, and we estimate θ by $\hat{\theta} = \overline{x} + 1/n$.

(a) Compute $\text{Bias}_\theta(\hat{\theta})$.

(b) Compute $\text{Var}_\theta(\hat{\theta})$.

(c) Compute $MSE_\theta(\hat{\theta})$.

(d) Is $\hat{\theta}$ a consistent estimator for θ? Explain.

5. Suppose P_θ has mean θ and variance 1, and we observe $x_1, \ldots, x_n$, and we estimate θ by $\hat{\theta} = \overline{x} + W_n$, where W_n is a random variable independent of the x_i with $P[W_n = n] = P[W_n = -n] = 1/n$ and $P[W_n = 0] = 1 - 2/n$. Repeat parts (a) through (d) of the previous question.

6. Consider the statistical model where $S = \{0, 1, 2, \ldots\}$, $\Omega = (0, \infty)$, and for $\theta \in \Omega$, P_θ is the Poisson(θ) distribution, so $P_\theta(s) = e^{-\theta}\theta^s \,/\, s!$. Suppose we observe the observations 11, 5, 8.5, and 7.5. Compute (with explanation) the Method-of-Moments Estimate of θ.

7. Consider the statistical model where $S = \Omega = \mathbf{R}$, and for $\theta \in \Omega$, $P_\theta = N(2, \theta)$, so that θ represents the variance (not the mean!). Suppose we observe the observations 3, -5, 6, and -2. Compute (with explanation) the Method-of-Moments Estimate of θ.

8. (Bayesian Inference) Suppose we have one of three dice: either six-sided (so the numbers $\{1, \ldots, 6\}$ are all equally likely), or eight-sided (so the numbers $\{1, \ldots, 8\}$ are all equally likely), or ten-sided (so the numbers $\{1, \ldots, 10\}$ are all equally likely), but we don't know which one we have. Thus, $\Omega = \{$six-sided, eight-sided, ten-sided$\}$. Suppose our prior distribution on Ω is given by π(six-sided) $= 4/7$, π(eight-sided) $= 1/7$, and π(ten-sided) $= 2/7$. Compute the posterior distribution for the unknown $\theta \in \Omega$, for each of the following cases:

(a) We observe just one roll, and it is a 4.

(b) We observe two rolls, and they are 4 and 5.

(c) We observe two rolls, and they are 4 and 7.

9. Let $\Omega = \{1, 2, 3\}$, with $P_1\{5\} = 1/2$, $P_1\{8\} = 1/3$, $P_1\{9\} = 1/6$, $P_2\{10\} = 1/2$, $P_2\{13\} = 1/3$, $P_2\{14\} = 1/6$, $P_3\{25\} = 1/2$, $P_3\{28\} = 1/3$, $P_3\{29\} = 1/6$. Suppose we observe $X_1, \ldots, X_n$. Let $D = \overline{X} - X_1$. Prove that D is ancilliary.