RATES OF CONVERGENCE FOR GIBBS SAMPLER AND OTHER MARKOV CHAINS

A thesis presented

by

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ABSTRACT

This thesis considers the convergence of Markov chains. It is particularly concerned with the question of how long a given Markov chain must be run until it is close to its stationary distribution. Sharp answers to this question are obtained for a wide variety of Markov chains, including several different random walks on groups, and certain versions of the Data Augmentation and Gibbs Sampler algorithms as used in Bayesian Statistics.

In the first part of the thesis, random walks on finite and compact Lie groups are considered. They are analyzed using harmonic analysis. In particular, we analyze a process of random rotations on the orthogonal group. The Weyl Character Formula allows us to obtain useful formulas for the irreducible characters of the group, and these formulas are then used to get bounds on the variation distance to normalized Harr measure after k random rotations. We prove the existence of a "cut-off phenomenon" (as defined by Aldous and Diaconis) for this process. This is the first such result on a non-finite group. In addition, we consider certain families of random walks on circle groups, and prove a fairly general theorem concerning their convergence to the uniform distribution.

In the second part of the thesis, we analyze the Data Augmentation and Gibbs Sampler algorithms. By using ideas related to Harris Recurrence for Markov chains, we obtain convergence rates for these processes in various cases. In particular, we obtain sharp results about the convergence rates when using Data Augmentation on finite sample spaces, and also when using Gibbs Sampling with Variance Component Models. The results give an indication of how long these iterative procedures need to be run until they converge. This is a question of great importance and interest to Statisticians.

We conclude with some directions for further research.

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