Supplement to

"Minorization conditions and convergence rates for Markov chain Monte Carlo"

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This page discusses two slight generalizations of Theorem 12 in the above paper. It is too late to include them in the published version, hence I mention them here. I thank Charlie Geyer, Gareth Roberts, and Richard Tweedie for helpful comments.

Using Theorem 5, it is very easy to generalize Theorem 12 in various ways, e.g. to values of k_0 other than 1. Setting $h(x,y) = 1 + MV(x) + MV(y)$, $j = rk$, and $C = V_d = \{x \in \mathcal{X}\,;\, V(x) \le d\}$, Theorem 5 immediately implies

Theorem 12′. *Suppose for some $V : \mathcal{X} \to \mathbf{R}^{\ge 0}$, some $\lambda < 1$ and $b < \infty$, some $\epsilon > 0$, some probability measure $Q(\cdot)$ on $\mathcal{X}$, and some $d > \frac{2b}{1-\lambda}$, we have $E(V(X^{(1)}) \mid X^{(0)} = x) \le \lambda V(x) + b$ for all $x \in \mathcal{X}$, and also $P^{k_0}(x,\cdot) \ge \epsilon Q(\cdot)$ for all $x \in V_d$. Then for any $0 < r < 1$ and $M > 0$, we have*

$$\|\mathcal{L}(X^{(k)}) - \pi\|_{\mathrm{var}} \;\le\; (1-\epsilon)^{[rk/k_0]} + (\alpha A)^{-1}\left(\alpha^{-(1-rk_0)}A^r\right)^k \left(1 + \frac{Mb}{1-\lambda} + ME_\nu(V(X_0))\right),$$

where

$$\alpha^{-1} \;=\; \frac{1 + 2Mb + \lambda Md}{1 + Md} \;<\; 1\,; \qquad A \;=\; 1 + 2(\lambda Md + Mb)\,.$$

For comparison to the work of Meyn & Tweedie (1993b), it is appropriate in the drift condition to assume that $V(x) \ge 1$ for all $x \in \mathcal{X}$, and that the constant term b is multiplied by the indicator function $\mathbf{1}_{V_d}$. Setting $h(x,y) = \frac{1}{2}(V(x) + V(y))$, this allows us to somewhat relax the condition on d, and leads to

Theorem 12″. *Suppose for some $V : \mathcal{X} \to \mathbf{R}^{\ge 1}$, some $\lambda < 1$ and $b < \infty$, some $\epsilon > 0$, some probability measure $Q(\cdot)$ on $\mathcal{X}$, and some $d > \frac{b}{1-\lambda}$, we have $E(V(X^{(1)}) \mid X^{(0)} = x) \le \lambda V(x) + b\mathbf{1}_{V_d}(x)$ for all $x \in \mathcal{X}$, and also $P^{k_0}(x,\cdot) \ge \epsilon Q(\cdot)$ for all $x \in V_d$. Then for any $0 < r < 1$, we have*

$$\|\mathcal{L}(X^{(k)}) - \pi\|_{\mathrm{var}} \;\le\; (1-\epsilon)^{[rk/k_0]} \;+\; (\alpha A)^{-1}\left(\alpha^{-(1-rk_0)}A^r\right)^k \,\frac{1}{2}\left(\frac{b}{1-\lambda} + E_\nu(V(X_0))\right),$$

where

$$\alpha^{-1} \;=\; \frac{b + \lambda d}{d} \;<\; 1\,; \qquad A \;=\; \lambda d + b\,.$$