# Errata for the SECOND EDITION of "A First Look at Rigorous Probability", by Jeffrey S. Rosenthal, World Scientific Publishing Co., 2006.

(Note: throughout, "line -x" means x lines from the bottom.)

#### Errata to Eighth Printing, 2020:

[With thanks to Damir Ferizović, David Scott, Daniel de Souza Severo, and Robert Zimmerman.

- p. 31, line 6: " $f_2 \mathbf{1} I_2$ " should be " $f_2 \mathbf{1}_{I_2}$ ".
- p. 40, Exercise 3.6.16 (c)–(e): "[...]" should be the floor function " $\lfloor ... \rfloor$ " (three times).
- Page 103, proof of Theorem 9.1.1: Note that by replacing  $X_n$  by  $X_n C$ , it suffices to assume that  $X_n \ge 0$ .
- Page 108, statement of Theorem 9.3.4: Assume that  $\epsilon > 0$ .
- Page 113, Exercise 9.5.8 line 2: " $F_t(\{1\})$ " should be " $F_t(1)$ ", and " $F_t(\{2\})$ " should be " $F_t(2)$ ".
- Page 126, line 1: " $E(|X|^k)$ " should be " $\mathbf{E}(|X|^k)$ ".
- Page 127, line -4: The double-integral is unnecessarily repeated twice.
- Page 127, line -1: "+  $\int_0^\infty$ " should be "-  $\int_0^\infty$ ".
- Page 134: Missing a final closing bracket at the end of the proof of Theorem 11.2.2.
- Page 145, line -1: "Theorem 12.1.4" should be "Lemma 12.1.4".
- Page 159, Exercise 13.4.13: " $\sum_{j \in \mathcal{X}}$ " should be " $\sum_{j \in S}$ " (twice).
- Page 173 line 5: "Billingley" should be "Billingsley" (of course!).
- Page 185, Exercise 15.3.12: Omit the initial "(a)".
- Page 187 line 6: expand "shrunk" to "space has been shrunk".

### Errata to Seventh Printing, 2016 (to be corrected in Eighth Printing, 2020):

[With thanks to Damir Ferizović, Daehyun Kim, and Martin Yang.]

- Page 33, line 7: after "for fixed Borel  $S \subseteq \mathbf{R}$ ", add "with  $\mathbf{P}(Y \in S) > 0$ ".
- Page 36 middle: replace the second " $\sum_{k=1}^{\infty} \mathbf{P}(B_{2k})$ " with " $\sum_{k=1}^{\infty} (1/4)$ ".

- Page 40, Exercise 3.6.18, line 2: better to change "lim" to "lim inf" to avoid issues of the *existence* of the limit. (Alternatively, the current definition of  $S_x$  can be taken to mean the limit *exists* and is  $\leq x$ .)
- Page 53, Exercise 4.5.10, first line: replace "i.i.d." with "independent", to make the statement stronger, and also because i.i.d. hasn't been introduced yet.
- Page 76, equation (7.2.2): should end with a comma, not a period.
- Page 77, lines 11 and 12 (twice): replace the word "obtaining" with the word "reaching" (for greater clarity).
- Page 85, line -6: strictly speaking, " $\{I_{ij}^{(1)}\}_{i,j\in S}$ " should be " $\{I_{ij}^{(1)}\}_{j\in S}$ ".

#### Errata to Sixth Printing, 2013 (to be corrected in Seventh Printing, 2016):

[With thanks to Anthony Brooms, Geoffrey Brown, Julian Ziegler Hunts, Owen Lyne, Byron Schmuland, Adrian Torchiana, and Peter Tye.]

- Page 3 middle, change "not used elsewhere in this book" to "not required elsewhere in this book".
- Page 5 middle, change "mathematical rigorous" to "mathematically rigorous".
- Page 21, line 8: " $A \subseteq \mathcal{F}$ " should be " $A \in \mathcal{F}$ ".
- Page 23, line -3: "a algebra" should be "an algebra".
- Page 29, eqn (3.1.2): "X(w)" should be " $X(\omega)$ ".
- Page 40, Exercise 3.6.13, line 2: "P" should be " $\mathbf{P}$ ", and "X" should be " $X_n$ ", and "for all i" should be "for all n".
- Page 46: omit the last two sentences (since we are already assuming that the  $X_n$  are non-negative).
- Page 57, line -4: " $\mu_y$ " should be " $\mu_Y$ ".
- Page 87, second half: "Sterling" should be "Stirling" (twice).
- Page 71, Exercise 6.3.1: Replace the first sentence by, "Let  $\mu$  have density  $4x^3 \mathbf{1}_{0 < x < 1}$ , and let  $\nu$  have density  $\frac{1}{2}x \mathbf{1}_{0 < x < 2}$ ."
- Page 119, lines 1–2: since Y is actually defined as a function on (0, 1), it is better to replace " $m(Y(n+1) Y(n)) < \infty$  discontinuities of size  $\geq 1/m$  within the interval (n, n+1]" with " $m[Y(1-\frac{1}{n}) Y(\frac{1}{n})] < \infty$  discontinuities of size  $\geq 1/m$  within the interval  $(\frac{1}{n}, 1-\frac{1}{n})$ ".
- Page 121, Exercise 10.3.2, line 2: omit the comma just before the period.

- Page 134: the last three lines of the proof of Theorem 11.2.2 are poorly phrased, since  $\phi_n(t)$  might be complex, and should be replaced by e.g.: " $n \to \infty$ . This means that for any  $\epsilon > 0$ , for sufficiently large n we have  $|nq_n| \leq \epsilon$ . Hence, as  $n \to \infty$  the limit is multiplied by  $e^{nq_n}$ , which becomes arbitrarily close to 1 and hence can be ignored."
- Page 147, lines 17–18: " $\mu$ " and " $\nu$ " should be interchanged (twice).
- Page 178, 2/3 of the way down the page: "only if" and "if" should be interchanged.
- Page 181, Exercise 15.2.7: change "a Markov chain" to "an irreducible Markov chain".
- Page 183, Exercise 15.3.3, line 3: change " $A_0$ " to " $A_1$ ".
- Page 187, line -12: " $\frac{1}{n} \mathbf{E} \left( Y_{\lfloor \underline{sn} \rfloor}^{(n)} Y_{\frac{\lfloor \underline{sn} \rfloor}{n}}^{(n)} Y_{\frac{\lfloor \underline{sn} \rfloor}{n}}^{(n)} \right) = \mathbf{E} \left( (Z_1 + \ldots + Z_{\lfloor \underline{sn} \rfloor}) (Z_1 + \ldots + Z_{\lfloor \underline{tn} \rfloor}) \right)$ should be " $\mathbf{E} \left( Y_{\frac{\lfloor \underline{sn} \rfloor}{n}}^{(n)} Y_{\frac{\lfloor \underline{tn} \rfloor}{n}}^{(n)} \right) = \frac{1}{n} \mathbf{E} \left( (Z_1 + \ldots + Z_{\lfloor \underline{sn} \rfloor}) (Z_1 + \ldots + Z_{\lfloor \underline{tn} \rfloor}) \right)$ ".
- Page 192, Exercise 15.6.8, line 2: change " $\int_0^t b B_s ds$ " to " $\int_0^t b dB_s$ ".

#### Errata to Fifth Printing, 2011 (corrected in Sixth Printing, 2013):

[After teaching from this book again after a five-year break, and also with thanks to Daniel Firka, Julian Ziegler Hunts, Jianlin Zou, Luis Mendo Tomás, and Sebastiaan Janssens.]

- Page 19, Exercise 2.5.6: change "finite unions" to "finite disjoint unions", and note that **P** is extended to  $A_n$  by finite additivity.
- Page 20: for clarity, replace "distributions" by "measures" (twice).
- Page 22, line 2: change "finite unions" to "finite disjoint unions".
- Page 25, line 1: "setset" should be "subset".
- Page 35, line 4: "considering" should be "consider".
- Page 36, line 9: missing "}".
- Page 36, several places: omit extra  $\{\ldots\}$ , e.g. " $\{H_n \cap H_{n+1}\}$ " should be simply " $H_n \cap H_{n+1}$ ", etc.
- Page 39, Exercise 3.6.5: for clarity, assume  $\mathcal{F} = 2^{\Omega}$ .
- Page 39, line 4: "P" should be "**P**".
- Page 40, Exercise 3.6.11: the notation "~" has not yet been introduced, so " $X_n \sim$  Uniform  $(\{1, 2, \ldots, n\})$ " should be replaced by " $\mathbf{P}(X_n = i) = 1/n$  for  $i = 1, 2, \ldots, n$ ".
- Page 40, Exercise 3.6.13: should be moved to LATER (since it uses expectation), e.g.

as Exercise 4.5.16. (And "E" should be "E", twice.) It could be replaced by e.g.

Let  $X_1, X_2, \ldots$  be defined jointly on some probability space  $(\Omega, \mathcal{F}, P)$ , with  $\sum_{i=1}^{\infty} i^2 \mathbf{P}(i \le X < i+1) \le C < \infty$  for all *i*. Prove that  $\mathbf{P}[X_n \ge C < \infty]$  $[n \ i.o.] = 0.$ 

- Page 46, statement of Theorem 4.2.2: assume the  $X_n$  are non-negative, and then omit " $\mathbf{E}(X_1) > -\infty$ " (since we haven't yet defined expected values of general random variables).
- Page 46, first line of proof: for greater clarity, replace "(3.1.6)" by "Proposition 3.1.5.(iii)". (Similarly page 107, line 5.)
- Page 46, lines -3 and -2: "E" should be boldface (twice).
- Page 49, exercise 4.3.3(a): change " $Z^+$  and  $Z^-$ " to " $Z^+ Z^-$ ".
- Page 54, Exercise 4.5.13(d): replace " $\mathbf{E}(X) < \infty$ " by " $0 < \mathbf{E}(X) < \infty$ ".
- Page 58, Lemma 5.2.1: note that the converse also holds.
- Page 60: introduce the abbreviations "WLLN" and "SLLN".
- Page 65, Exercise 5.5.9, Hint: specify that y > 0 for the first part, too.
- Page 66, Exercise 5.5.13, Hint: "r different sums" should be "r + 1 different sums".
- Page 71, Exercise 6.3.1 is a repeat of Exercise 4.5.1 (page 52), and should be replaced by e.g.:
  - Let  $\mu$  have density  $x^3 \mathbf{1}_{0 < x < 1}$ , and let  $\nu$  have density  $x \mathbf{1}_{0 < x < 2}$ .

  - (a) Compute  $\mathbf{E}(X)$  where  $\mathcal{L}(X) = \frac{1}{3}\mu + \frac{2}{3}\nu$ . (b) Compute  $\mathbf{E}(Y^2)$  where  $\mathcal{L}(Y) = \frac{1}{6}\mu + \frac{1}{3}\delta_2 + \frac{1}{2}\delta_5$ . (c) Compute  $\mathbf{E}(Z^3)$  where  $\mathcal{L}(Z) = \frac{1}{8}\mu + \frac{1}{8}\nu + \frac{1}{4}\delta_3 + \frac{1}{2}\delta_4$ .
- Page 76, Exercise 7.2.5(d): for clarity, prepend "Use the fact that s(c) = 1 to".
- Page 85, statement of Theorem 8.1.1, displayed eqn: " $nu_{i_0}$ " should be " $\nu_{i_0}$ ".
- Page 114, line -1: " $\beta^{\alpha}$ " should be " $\beta^{-\alpha}$ ".
- Page 115, line 1: " $t^{x-1}$ " should be " $t^{\alpha-1}$ ".
- Page 117, line 2: for clarity, change "distributions" to "measures".
- Page 125, middle of page, displayed equations: final inequality ( $\leq$ ) is actually an equality (=).
- Page 144, line -5: omit the second "then".
- Page 159, Exercise 13.4.5: Assume  $\mathbf{P}\{1\} = \mathbf{P}\{2\} = \mathbf{P}\{3\} = 1/3$ .

- Page 159, Exercise 13.4.6: Change " $\mathbf{E}(X | \mathcal{G})$  and  $\mathbf{E}(Y | \mathcal{G})$  need not be independent" to "we might have  $\mathbf{E}(XY | \mathcal{G}) \neq \mathbf{E}(X | \mathcal{G}) \mathbf{E}(Y | \mathcal{G})$ ".
- Page 165, line 3: "Theorem 14.1.3" should be "Theorem 14.1.5".
- Page 171: for clarity, replace "for a submartingale, the same inequality holds" by "for submartingales, similarly inequalities hold".
- Page 172, line -2: "Then follows" should be "It then follows".
- Page 173, Exercise 14.4.5: Assume  $\mathbf{E}|Z| < \infty$ .
- Page 181, Theorem 15.2.3: " $\rightarrow 0$ " should be "= 0".
- Page 187: O(1/n) should be  $O(1/\sqrt{n})$  (three times).
- Page 192: Exercise 15.6.7: For greater generality, replace " $X_t = at + bB_t$ " by " $X_t = X_0 + at + bB_t$ " in part (a), and " $N(at, b^2t)$ " by " $N(X_0 + at, b^2t)$ " in part (b).
- Page 192: Exercise 15.6.8: Replace " $\int_0^t s \, ds$ " by " $\int_0^t a \, ds$ ".
- Page 206 middle, "it suffice to assume" should be "it suffices to assume".

#### Errata to Fourth Printing, 2010 (corrected in Fifth Printing, 2011):

[With thanks to David Alexander, Martin Hazelton, Andrea Lecchini-Visintini, Gareth Roberts, Igal Sason, Mohsen Soltanifar, and Albert Zevelev.]

- p. 13, line 3 of proof of Lemma 2.3.11: " $\mathcal{B}_m^C$ " should be " $B_m^C$ ".
- p. 23, Exercise 2.7.3, part (a): "an semialgebra" should be "a semialgebra".
- p. 31, line 5: " $f^{-1}((\infty, x])$ " should be " $f^{-1}((-\infty, x])$ ".
- p. 44, line 1, and again on line 19: " $\sum_j y_j \mathbf{1}_{B_i}$ " should be " $\sum_j y_j \mathbf{1}_{B_j}$ ".
- p. 48, line -5: "non non-negative" should be "not non-negative".
- p. 66, line -8: "Holder" should be "Jensen".
- p. 69, first line of Proposition 6.2.1: " $\mu_I$ " should be " $\mu_i$ ".
- p. 74, line 7: "F(z) for each  $z \in \mathbf{R}$ " should be "F(x) for each  $x \in \mathbf{R}$ ".
- p. 85, displayed equation in Theorem 8.1.1: " $X_{\pm i_1}$ " should be " $X_1 = i_1$ ".
- p. 106, line 15: "integral" should be "integrable".
- p. 106, line 17: " $\mathbf{E}(X)$ " should be " $\mathbf{E}(X_0)$ ".
- p. 106, line -1: "E(Y)" should be " $\mathbf{E}(Y)$ ".

- p. 136, Exercise 11.3.2: " $Y_n$ " should be " $Y_k$ " (twice).
- p. 137, conclusion of Theorem 11.4.1: " $\mu \Rightarrow \mu$ " should be " $\mu_n \Rightarrow \mu$ ".
- p. 146, line -6: " $mu_{sing}$ " should be " $\mu_{sing}$ ".
- p. 155, middle: " $\{ \{X_1 \leq a\} \cap \{X_2 \leq b\} : a, b \in \mathbf{R} \}$ " should be " $\sigma(\{ \{X_1 \leq a\} \cap \{X_2 \leq b\} : a, b \in \mathbf{R} \})$ ".
- p. 171, line 11: " $\mathbf{E}\left(\lim_{M\to\infty} U_M^{(\alpha,\beta)} = \infty\right) = \infty$ " should be " $\mathbf{E}\left(\lim_{M\to\infty} U_M^{(\alpha,\beta)}\right) = \infty$ ".
- p. 178, line -6: " $\{X_{t_1}, \ldots, X_{t_k}\} \in H\}$ " should be " $\{(X_{t_1}, \ldots, X_{t_k}) \in H\}$ ".
- p. 192, Exercise 15.6.8: replace " $Z_t = \exp[-(a+\frac{1}{2}b^2)t+X_t]$ " with " $Z_t = \exp[-2aX_t/b^2]$ ". (In fact, the first  $Z_t$  is also a martingale, but it is less useful than the second version.)
- p. 202, lines 14–15: to avoid subtleties about equivalent repeating decimals, perhaps replace " $c_i = 4$  if  $d_i = 5$ , while  $c_i = 5$  if  $d_i \neq 5$ " with " $c_i = 2$  if  $d_i \geq 5$ , while  $c_i = 7$  if  $d_i < 5$ ". (This is not strictly necessary, but it makes the argument a bit cleaner.)
- p. 204, Exercise A.3.8: change " $\sum_{i=1}^{\infty}$ " to " $\sum_{i=2}^{\infty}$ " (twice), and " $\int_{1}^{\infty}$ " to " $\int_{2}^{\infty}$ ".
- p. 204, Exercise A.3.9: change " $\sum_{i=1}^{\infty}$ " to " $\sum_{i=3}^{\infty}$ " (twice).
- p. 207, second line of Exercise A.5.1: "equivalence class" should be "equivalence relation".

#### Errata to Second Printing, 2007 (corrected in Fourth Printing, 2010):

[With thanks to Orn Arnaldsson, Bent Jørgensen, Chris Mansley, Kohei Nagamachi, Patrick Rabau, Mohsen Soltanifar, Hermann Thorisson.]

- p. 19, Exercise 2.5.6, and also p. 22, proof of Lemma 2.6.2: replace " $A_1, A_2, \ldots \in \mathcal{J}$ " by " $A_1, A_2, \ldots$  are finite unions of elements of  $\mathcal{J}$ ".
- p. 23, Exercise 2.6.4: " $\mathbf{P}(\emptyset) = 1$ " should be " $\mathbf{P}(\Omega) = 1$ ".
- p. 23, Exercise 2.7.3, part (b): interchange "semialgebra" and "algebra". (Also, for stylistic improvement, swap parts (a) and (b).)
- p. 30, last line of proof of Proposition 3.1.5: " $\{X \le x\}$ " should be " $\{Z \le x\}$ ".
- p. 33, line 7: second " $\mathbf{P}(X \in T)$ " should be " $\mathbf{P}(Y \in S)$ ".
- p. 39, Exercise 3.6.8: exercise is correct, but special cases like " $d \le b + c a$ " and "d > b + c a" should be modified.
- p. 40, Exercise 3.6.14: insert "independent" before "non-negative".

• p. 74, the proof of Lemma 7.1.2 is sloppily written and should be replaced by:

Since F is right-continuous, we have that  $\inf\{x; F(x) \ge u\} = \min\{x; F(x) \ge u\}$ , i.e. the infimum is actually obtained. It follows that  $\phi(u) \le x$  if and only if  $u \le F(x)$ . Hence, since  $0 \le F(x) \le 1$ , we obtain that  $\mathbf{P}(\phi(U) \le x) = \mathbf{P}(U \le F(x)) = F(x)$ .

- pp. 113–114: "Example 9.5.9" should be "Exercise 9.5.9", and similarly for 9.5.11 and 9.5.12.
- p. 114, Exercise 9.5.12: "characteristic function" should be "moment generating function".
- p. 118 middle, the three lines following Figure 10.1.2: replace "g" by "f" (five times).
- p. 118, line -5: " $F(w) \ge b$ " should be " $F(z) \ge b$ ".
- p. 121, Exercise 10.3.5: "four conditions" should be "five conditions".
- p. 126, statement of Lemma 11.1.2: " $\phi(t)$ " should be " $e^{itx}$ ".
- p. 128, line -7: " $\pi$ " should be removed from the equation.
- p. 131, line 9: "lim" should be "lim" (twice); " $F_n$ " should be " $F_{n_k}$ " (twice); and " $\mu_n$ " should be " $\mu_{n_k}$ ".
- p. 136, lines 3–4: the sentence in brackets is somewhat misleading and should be revised or omited.
- p. 136, eqn (11.3.1): " $\mathbf{1}_{|Z_{nk} \ge \epsilon s_n}$ " should be " $\mathbf{1}_{|Z_{nk}| \ge \epsilon s_n}$ "
- p. 166, lines 6–7: "bets \$1 on tails, then if they win they bet \$2 on heads" should be "bets \$1 on heads, then if they win they bet \$2 on tails".
- p. 173, Exercise 14.4.1: should say " $\mathbf{P}(Z_i = 1) = \mathbf{P}(Z_i = 0) = 1/2$ ", and " $X_1 = 2Z_1 1$ ".
- p. 174, Execise 14.4.12(a), Hint: " $\mathbf{P}(\tau \ge 3m)$ " should be " $\mathbf{P}(\tau > 3m)$ ".

#### Errata to First Printing, 2006 (corrected in Second Printing, 2007):

[With thanks to Joe Blitzstein, Saad Siddiqui, Emil Zeuthen.]

- p. 9, line -10: "all intervals" should be "all subsets".
- p. 18, eqn. (2.5.2): "P(B)" should be "P(B)".
- p. 19, the last sentence in the proof of corollary 2.5.4 is questionable (since we may have  $D_n \notin \mathcal{J}$ ), and should be replaced by:

It then follows from (2.5.5) that

$$\mathbf{P}\left(\bigcup_{n} B_{n}\right) = \mathbf{P}\left(\bigcup_{n} D_{n}\right) = \mathbf{P}\left(\bigcup_{n} \bigcup_{i=1}^{k_{n}} J_{ni}\right) = \sum_{n} \sum_{i=1}^{k_{n}} \mathbf{P}(J_{ni}).$$

On the other hand,

$$B_n = \bigcup_{m \le n} \bigcup_{i=1}^{k_m} (J_{mi} \cap B_n)$$

and the union is disjoint, with  $J_{ni} \subseteq B_n$ , so

$$\mathbf{P}(B_n) = \sum_{m \le n} \sum_{i=1}^{k_m} \mathbf{P}(J_{mi} \cap B_n) \ge \sum_{i=1}^{k_n} \mathbf{P}(J_{ni} \cap B_n) = \sum_{i=1}^{k_n} \mathbf{P}(J_{ni}),$$

and hence

$$\sum_{n} \mathbf{P}(B_n) \geq \sum_{n} \sum_{i=1}^{k_n} \mathbf{P}(J_{ni}) = \mathbf{P}\left(\bigcup_{n} B_n\right).$$

- p. 20, eqn. (2.5.10): " $(\infty, x]$ " should be " $(-\infty, x]$ ".
- p. 22, line -10: " $P_1$ " should be " $\mathbf{P}_1$ ", and " $P_2$ " should be " $\mathbf{P}_2$ ".
- p. 151, first line of Section 13.1: "We being" should be "We begin".
- p. 162, last line: " $X_n = 5$ " should be " $X_n = -5$ ".
- p. 205, Exercise A.4.5: "contraction" should be "contradiction".
- p. 206, line 4: "g(x)/h(x)" should be "|g(x)/h(x)|".
- p. 206, line 7: "limsup" should be "lim".

## ERRATA FOR THE ONLINE SOLUTIONS FILE:

[With thanks to Danny Cao, Felix Pahl, Byron Schmuland, and David Scott.]

[CORRECTED AND UPDATED by Xinyu Huo, August 2023.]

- Exercise 2.7.14(a): "for all  $a \neq b$ " should be "for all  $\{a, b\} \in \mathcal{J}$ ".
- Exercise 2.7.22(a): add a description of  $\Omega$ , and extend  $\mathcal{F}$  to allow for separate subsets A and B, not just a single subset A.

- Exercise 3.6.6(a): solution is not quite correct (and will be revised).
- Exercise 3.6.16(c): in the displayed equation,  $\sum_{n=1}^{\infty} (\frac{1}{n})^{2n}$  should be " $2 \sum_{n=1}^{\infty} (\frac{1}{n})^{2n}$ .
- Exercise 3.6.18: in the first displayed equation,  $\cap_{s=1}^{\infty} \bigcup_{N=m}^{\infty} \bigcap_{n=N}^{\infty} \{\frac{1}{n} \sum_{i=m}^{n} \mathbf{1}_{A_i} \le x + \frac{1}{s}\}$ " should instead be  $\cap_{s=1}^{\infty} \bigcap_{N=m}^{\infty} \bigcup_{n=N}^{\infty} \{\frac{1}{n} \sum_{i=m}^{n} \mathbf{1}_{A_i} \le x + \frac{1}{s}\}$ ".
- Exercise 4.5.10: solution is not correct, and should be replaced by the one at: http://probability.ca/jeff/ftpdir/Ex4.5.10sol.pdf Also, in the first expression for  $\mathbf{E}(S^2)$ , omit the factor of "2".
- Exercise 7.4.6: In the displayed formula for  $s_c(a)$ , each " $W_n$ " should be " $W_1$ ", and each " $S_c$ " should be " $s_c$ ".
- Exercise 8.5.8: In lines 4 and 5, the second " $P_i(\cdots)$ " should be " $\mathbf{P}(\cdots \mid X_1 = k)$ ".
- Exercise 8.5.10(a): Replace all sentences after the first with: "So,  $\mathbf{P}_i(X_n = i_0 \ i.o.) > 0$ , i.e.  $\lim_{k\to\infty} \mathbf{P}_i(\#\{n \ge 1 : X_n = i_0\} \ge k) > 0$ . So, from Lemma 8.2.2,  $\lim_{k\to\infty} f_{ii_0}(f_{i_0i_0})^{k-1} > 0$ . This is only possible if  $f_{i_0i_0} = 1$ . Hence,  $i_0$  is recurrent."
- Exercise 8.5.20(c): this is not quite correct as stated, since the chain might not be irreducible even on the state spaces  $S_r$ . But it is still true that there is at least one stationary distribution on each  $S_r$ , hence at least two stationary distributions in total.
- Exercise 9.5.10, last line: "b" should be "s" (twice).
- Exercise 9.5.14(a): answer should be 1, not  $\infty$ .
- Exercise 13.4.6: solution is incorrect (and will be revised).
- Exercise 13.4.10: solution is incorrect (and will be revised). For a better example, let  $\mathbf{P}(X = 1, Y = 1) = \mathbf{P}(X = 1, Y = -1) = \mathbf{P}(X = -1, Y = 2) = \mathbf{P}(X = -1, Y = -2) = 1/4$ . Then X and Y are clearly not independent, but  $\mathbf{E}(Y|X) = 0 = \mathbf{E}(Y)$ .