Supplement for the end of Section 4.4 (page 132) of

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by Jeffrey S. Rosenthal

(4.4.s1) Remark. We assumed above that the generator G was *bounded*, i.e. $\sup_{i,j \in S} |g_{ij}| < \infty$. If this condition is violated, then strange behaviour can occur[1]. For example, suppose $S = \{1, 2, 3, \ldots\}$, with $g_{i,i+1} = 2^i$ and $g_{ii} = -2^i$ for all $i \in S$, and $g_{ij} = 0$ otherwise. Then this process can only *increase*, by 1 each time. Furthermore, the expected time to increase from i to $i+1$ is equal to $1/g_{i,i+1} = 2^{-i}$. Hence, starting from $X_0 = 1$, the expected time to increase all the way to infinity is equal to $\sum_{i=1}^{\infty} 2^{-i} = 1$. That is, in a finite time (equal to one second, on average), the process will escape all the way to infinity, and hence "vanish". This property is called being *explosive*. Clearly, for such processes, formulas for $P^{(t)}$ such as (4.4.7) no longer hold.

[1] I thank Tom Salisbury for discussing this issue with me.